RAPPORT
Methods for Accuracy Verification of Positioning Module
Borlänge
FOI-projekt 5148
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## Acronyms

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<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AO</td>
<td>Atom Optic</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth-Centred-Earth-Fixed</td>
</tr>
<tr>
<td>FOG</td>
<td>Fiber Optic Gyro</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>IFOG</td>
<td>Interferometric Fiber-Optic Gyros</td>
</tr>
<tr>
<td>IOG</td>
<td>Integrated Optics Gyro</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
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<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
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<tr>
<td>MEMS</td>
<td>Microelectromechanical Systems</td>
</tr>
<tr>
<td>MMS</td>
<td>Mobile Mapping System</td>
</tr>
<tr>
<td>RLG</td>
<td>Ring Laser Gyro</td>
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<tr>
<td>SAW</td>
<td>Surface Acoustic Wave</td>
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</table>
Introduction

This report was written as a part of project “Utveckling mobil datafångst” FOI-projekt 5148 carried on at Royal Institute of Technology (KTH) in Stockholm. The mobile data collection has received much attention from researchers all over the world during the past 15 – 20 years. The result of this extensive work is a number of commercially available products, usually called mobile mapping systems (MMS), which are used for collection of geospatial data from cars or aircrafts. The heart of each MMS is a positioning module based on combination of GNSS receiver and inertial navigation system (INS). The module determines the position and orientation of the platform. The platform is equipped with a mapping sensor, usually a laser scanner and/or one or more digital cameras. The accuracy of collected data greatly depends on the accuracy of the positioning module. One of the main goals of this report is to assess the quality of the positioning module.

Traditionally, the inertial navigation has not been used for surveying purposes and hence the basic principles and error sources are not generally known among the surveyors. For this reason the report includes also a discussion on this topic.

Inertial navigation

Introduction to Inertial Navigation Systems (INS)

Based upon the dead reckoning concept, an INS tracks the changes in position, velocity and orientation in a defined reference frame by utilizing a processor and an inertial measurement unit (IMU). Its operation is based upon Newton’s law of motion, where every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it. IMU usually consists of three orthogonally mounted accelerometers and gyroscopes. An accelerometer measures so called specific force, i.e. the force induced by the acceleration of the platform and by the gravity. It detects the magnitude of the specific force along its sensitive axis. Please note that output from accelerometers is given in units of acceleration (m/s²) but still it is denoted as “specific force”. In order to determine the acceleration of the platform, the gravity acceleration projected on the sensitive axis must be subtracted from the output. The gravity acceleration vector is obtained from a model and the orientation of the sensitivity axis of the accelerometer is determined by gyroscopes. A gyroscope measures rotation rate around its sensitivity axis. The rotation is determined relative to the inertial frame, i.e. the gyroscopes sense both the rotation of the Earth and the rotation of the platform with respect to the Earth. This allows the system to update the information of which direction it is facing relative to its initial orientation.

In order to obtain navigation solution, the IMU measurements need to be transformed from its operating frame (body frame) to a chosen reference frame, which is typically the Earth-Centred-Earth-Fixed (ECEF) or local horizontal navigation frame. A processor is utilized to continuously integrate the output from the sensors to compute the change in position, velocity and orientation.
expressed in the chosen navigation frame. Combination of IMU and processor is usually called INS. While the underlying principles of INS operation are common, their implementation may take a variety of different forms (Titterton and Weston, 2004). The next subsection discusses the two platforms used when employing INS.

**INS Platform**

Inertial navigation is a relative positioning technique in which an inertial measurement unit (IMU) is tracked relative to its initial starting point. An IMU usually consists of three orthogonal accelerometers and three gyroscopes that are aligned with the accelerometers, as shown in Figure 1. An inertial navigation system (INS) consists of an IMU together with a navigation processor, which uses measurements from the IMU in order to track its orientation, position and velocity in some frame of reference in which they are desired.

![Figure 1. (a) A strapdown IMU. (b) A stable platform INS (Woodman, 2010)](image)

Practical inertial navigation systems may take a variety of forms. These forms generally fall into one of the two basic categories:

- strapdown systems (Figure 1a)
- stable platform systems (Figure 1b)

Although the two types of system are very different, both physically and computationally, it must be stressed that the underlying principle of double integrating acceleration in the reference frame is the same.

In stable platform systems, to keep track of orientation the angles between adjacent gimbals are read using angle pick-offs. To track position the three accelerometer signals are double integrated, see Figure 2.
In a strapdown platform system, to track the orientation the signals from the rate gyroscopes are integrated (see Figure 3). To calculate the position of the device the accelerometer signals are resolved into global coordinates using the known orientation, as determined by the integration of the gyro signals.

Strapdown systems have reduced mechanical complexity and as a result tend to be physically smaller and lighter than stable platform systems. These benefits are achieved at the cost of increased computational complexity; however, the requirements are now trivial relative to the computational power of modern processors. As a result strapdown systems have become the dominant type of INS.

**Inertial Sensor Errors**

As all other measuring sensors, the accelerometers and gyroscopes are not “perfect”, error-free sensors. The error in measured specific force is usually called bias and corresponding error for gyroscopes is called drift. There are number of factors influencing the “correctness” of the output, e.g. manufacturing limitations, temperature and magnetic field variations, acceleration induced errors etc. The behaviour of the sensor error is difficult to model exactly, but it is possible to describe it stochastically. The Allan Variance technique is a mathematical method which allows a user to assess the real performance of any gyro in relation to the dynamics of his application, taking into account the effects.
of bias, noise, drift and long term sensor instability. Sensor error is always measured by averaging successive data samples - the challenge is to select a suitable time over which to average sampled data. Averages taken over short time intervals will be dominated by noise, those over a longer period by longer term drift. The technique involves selecting a range of time intervals (e.g., from 0.01s to 500s) over which to average data. The variation (standard deviation) from one averaged time period to the next is calculated and plotted against the averaging interval in log-log form. The resulting graph has a characteristic 'bathtub' shape. By examination of this graph, it is possible to compute the key defining characteristics of the gyro, namely angular random walk, bias instability and rate random walk.

The constant part of drift (bias) is the output from the sensor when it is NOT experiencing any rotation (acceleration). In a perfect world, one could make allowance for a fixed bias error. Unfortunately the "constant" error tends to vary, both with temperature and over time. It can be attributed to a number of components: calibration errors, switch-on to switch-on changes, variation over temperature, effects of shock (g level).

![Common input/output error types](http://www.siliconsensing.com/information/glossary)

**Figure 4.** Common input/output error types. The figure illustrates the errors in typical inertial sensors as a function of the input/output relationship. Bias (a) is any nonzero sensor output when the input is zero. Scale-factor error (b) often results from aging or manufacturing tolerances. Nonlinearity (c) is present in most sensors to some degree. Scale factor sign asymmetry (d) is often from mis-matched push-pull amplifiers. A dead zone (e) is usually due to mechanical stiction or lock-in (for example, in a ring laser gyro). Quantization error (f) is inherent in all digitized systems. Quantization error may not be zero mean when the input is held constant (Grewal, 2001).

**Bias** is the output that has no correlation with the input (see Figure 4). It can be asymmetric for positive and negative inputs and have an instability that is given by random variation if the output is computed over a specified sample interval.

---

The instability of bias is also called bias drift. Bias is normally expressed in [deg/hr] or [rad/s] for gyroscopes and [m/s²] or [mg] for accelerometers.

**Bias drift** refers specifically to the variation of the bias over time, assuming all other factors remain constant. Basically this is a warm-up effect, caused by the self heating of the gyro and its associated mechanical and electrical components. This effect would be expected to be more prevalent over the first few seconds after switch-on and to be almost non-existent after (say) five minutes.

**Scale factor** is the ratio of change in the output of the sensor with respect to true intended measurement (see Figure 4). It will normally be expressed in [ppm] (parts per million) for both gyroscopes and accelerometers. Like for bias, the scale factor can be asymmetric for positive and negative inputs. **Sensitivity** is related to scale factor. The difference is that sensitivity relates to a secondary input, e.g. change in temperature while scale factor relates to an intended primary input.

Both bias (not bias drift) and scale factor can be determined by calibration. In a calibration procedure, the bias and scale factor are determined by comparing known parameters e.g. the earth gravity or well-known angles to measured output.

**Bias (drift) Instability** is a fundamental measure of the 'goodness' of an accelerometer (gyro). It is defined as the minimum point on the Allan Variance curve, usually measured in m/s² (°/hr). It represents the best bias stability that could be achieved for a given sensor, assuming that bias averaging takes place at the interval defined at the Allan Variance minimum. This is the most important parameter for prediction of accuracy of unsupported inertial navigation.

**Angular (Velocity) random walk** is a measure of gyro (accelerometer) noise and has units of °/√h or °/√s (m/s/√s). It can be thought of as the variation (or standard deviation), due to noise, of the result of integrating the output of a stationary gyro over time. So, for example, consider a gyro with an ARW of °/√s being integrated many times to derive an orientation angle: For a stationary, east-west oriented gyro, the ideal result - and also the average result - would be zero. But the longer the integration time, the greater will be the spread of the results away from the ideal zero. Being proportional to the square root of the integration time, this spread would be 1° after 1 second and 10° after 100 seconds. This parameter is important for prediction of accuracy of supported inertial navigation.

Gyroscopes and accelerometers are often mounted in groups of three sensors that make up an orthogonal triad. Errors in orthogonality (also called axes misalignment) will result in errors of measurements as two sensors measure part of the same input. Axes misalignment can be calibrated or modelled in the INS error equation.

**Classification of IMUs/Inertial Sensors**

The average error that can be expected after a given duration depends on the quality of the IMU that is used. Several authors have categorized IMUs into
several grades, which broadly correspond to different inertial technologies, see Table 1.

Table 1. Specifications for different grades of IMU (Schwarz and El-Sheimy, 1999; Petovello, 2003); still valid 2012. (1 nmi = 1.852 km)

<table>
<thead>
<tr>
<th>Grade Performance</th>
<th>Strat Str</th>
<th>Nav Str</th>
<th>Tac Str</th>
<th>Auto Str</th>
<th>Cons Str</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standalone position errors</td>
<td>1 nmi/24hr</td>
<td>1 nmi/hr</td>
<td>&gt;10 nmi/hr</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gyroscopes</th>
<th>Bias [deg/hr]</th>
<th>Scale factor [ppm]</th>
<th>Noise (ARW) [deg/√hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0001</td>
<td>5 - 50</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Accelerometers</th>
<th>Bias [μg]</th>
<th>Scale factor [ppm]</th>
<th>Noise [μg/hr/√Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>10 - 20</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 5 gives lower bounds on the approximate drift for different grades of device. The results in the figure are obtained by a simulation that assumed a stationary device and considered only random error sources (Woodman, 2010). However, in practice, drift will grow more rapidly due to additional systematic and dynamic errors. The reason for this is because local gravitational conditions continually fluctuate due to tides, seismic activities and the gravitational pull of nearby celestial objects. The geophysical limit shown in Figure 5 is the growth of drift when using a perfect IMU but without accounting for such fluctuations (Woodman, 2010).
Figure 5. Approximate drift due to random error sources when using a commercial-grade, tactical-grade, navigation-grade, strategic-grade or “perfect” IMU (Woodman, 2010). Note that even a perfect IMU is not by itself sufficient to track position indefinitely without the accumulation of drift.

**Error Propagation in Inertial Navigation Systems**

Errors in the accelerations and angular rates lead to steadily growing errors in position and velocity components due to the integration (see the section above). This degradation of measurement accuracy propagates into the navigation solution (i.e. position and velocity) at rates dependent on the integrity of the component sensors, the algorithms employed, and the duration of the unaided navigation. It is for this reason that the INS is usually aided with either GNSS or some other augmentation sensor data.

For conventional inertial navigation systems, gyros play the most important role in terms of navigation accuracy. Among the errors of gyros, long-term bias instability is the dominant error when it works alone (without aiding from e.g. GNSS). Position drifts after a certain period of GNSS outage is often used as an indicator of the quality of INS navigation systems.

Bias instability is normally picked from technical specifications of sensors; otherwise, it can be estimated using the Allan Variance analysis (which is out of the scope of this report). For example, consider a MEMS (Microelectromechanical systems) gyro with a bias instability of 0.01° degrees per second. How much position drift will it cause after losing GNSS signals?

We have to make some assumptions in order to simplify the analysis: i) assume the vehicle is static, ii) assume that the system has no errors in position, velocity, and attitude at the moment of losing GNSS signals, iii) assume that the gyro bias error is constant during the GNSS outage.

Due to the assumption that the vehicle is not moving, the heading error caused by the gyro drift will not generate a position error. Consequently, we can focus on the errors along pitch and roll, that is, the tilt angles. This tilt error will cause a
component of acceleration due to gravity to be projected onto the horizontal axes and then be integrated twice to become a position drift (El-Sheimy and Niu, 2007):

\[
\Delta \theta = \int_0^t \Delta \omega \cdot dt = \Delta \omega \cdot t \\
\Delta v = \int_0^t (g \cdot \Delta \theta) \cdot dt = \int_0^t (g \cdot \Delta \omega \cdot \tau) \cdot d\tau = \frac{1}{2} \cdot g \cdot \Delta \omega \cdot t^2 \\
\Delta p = \int_0^t \Delta v \cdot d\tau = \int_0^t \left(\frac{1}{2} \cdot g \cdot \Delta \omega \cdot \tau^2\right) \cdot d\tau = \frac{1}{6} \cdot g \cdot \Delta \omega \cdot t^3
\]

(1) (2) (3)

where:

- t is the time after losing GNSS,
- \( \Delta \omega \) is the assumed constant gyro bias,
- \( \Delta \theta \) is the tilt error,
- \( \Delta v \) is the velocity error,
- \( \Delta p \) is the position error and
- g is the acceleration due to gravity.

A bias error in accelerometers \( \Delta a \) and gyroscopes \( \Delta \omega \) will introduce errors in velocity and position according to Table 2.

**Table 2. Error in velocity and position due to gyroscope and accelerometer bias**

<table>
<thead>
<tr>
<th></th>
<th>Error from accelerometer bias</th>
<th>Error from gyroscope bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity error</td>
<td>( \Delta v = \Delta a \cdot t )</td>
<td>( \Delta v = \frac{1}{2} \cdot \Delta \omega \cdot g \cdot t^2 )</td>
</tr>
<tr>
<td>Position error</td>
<td>( \Delta p = \frac{1}{2} \cdot \Delta a \cdot t^2 )</td>
<td>( \Delta p = \frac{1}{6} \cdot \Delta \omega \cdot g \cdot t^3 )</td>
</tr>
</tbody>
</table>

Assuming a GNSS signal outage of 30 seconds and substituting the gyro bias error of 0.01 deg/s into equation (3), the derived position drift is:

\[
\Delta p = \frac{1}{6} \cdot 9.81 \cdot \left(0.01 \cdot \frac{\pi}{180}\right) \cdot 30^3 = 7.7 \text{ m}
\]

(4)

The bias error contribution is linear, i.e. a gyro with 0.001 deg/s bias would contribute a position error of 0.77 m over 30 seconds navigation time. However, this is only the drift along one direction (north or east). Considering the two-dimensional error, the total position drift is:

\[
\sqrt{2} \cdot \Delta p = \sqrt{2} \cdot 7.7 \text{ m} = 10.9 \text{ m}
\]

(5)

Please note that this evaluation is based on a highly simplified analysis based on the three stated assumptions. The results will probably be too optimistic; however, it can be used as a rule of thumb.

In order to relate sensor biases to something more understandable Table 3 gives the position-error for a pure inertial sensor (current level of performance for MEMS sensors of today, cf. figures 14 and 17). Bias errors scale linearly, i.e. a 0.1
deg/hr gyro bias would contribute a position error of 0.17 m over 60 seconds of time.

### Table 3. Position errors due to sensor biases

<table>
<thead>
<tr>
<th>Bias</th>
<th>60 s</th>
<th>120 s</th>
<th>300 s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyro: 1 deg/hr</td>
<td>1.7 m</td>
<td>13.7 m</td>
<td>214 m</td>
</tr>
<tr>
<td>Acc: 50 μg</td>
<td>0.9 m</td>
<td>3.5 m</td>
<td>22 m</td>
</tr>
</tbody>
</table>

#### Mechanical Gyroscopes

A mechanical gyroscope makes use of the inertial properties of a wheel or rotor spinning at high speed. The wheel is mounted on two gimbals which allow it to rotate in all three axes, as shown in Figure 6. A spinning wheel tends to maintain the direction of its spin axes in space, i.e. it will resist changes in orientation. Hence, the wheel will remain at a constant global orientation and the angles between adjacent gimbals will change. To measure the orientation of the device the angles between adjacent gimbals can be read using angle pick-offs.

![Gyroscope diagram](image)

**Figure 6. A conventional mechanical gyroscope (Woodman, 2007)**

A mechanical gyroscope measures orientation; in contrast, nearly all modern gyroscopes (including the optical and MEMS types) are rate-gyros, which measure angular velocity. The main disadvantage of mechanical gyroscopes is that they contain moving parts. Moving parts cause friction, which in turn causes the output to drift over time.

#### Optical Gyroscopes

Optical gyroscopes work on the principle that, in a given medium, light travels at a constant speed. If light is sent in both directions around a non-rotating closed-loop waveguide made of mirrors or optical fiber, the path length is the same for
both beams (see Figure 7). However, if the waveguide is rotated about an axis perpendicular to its plane, then, from the perspective of an inertial frame, the reflecting surfaces are moving further apart for light travelling in the same direction as the rotation and closer together for light travelling in the opposite direction. Thus, rotating the waveguide in the same direction as the light path increases the path length and rotating it in the opposite direction decreases the path length (see Figure 7). This is known as the Sagnac effect.

![Figure 7. Effect of closed-loop waveguide rotation on path length](image)

Optical gyros contain no moving parts and require only a few seconds to start-up. The accuracy of an optical gyro is largely dependent on the length of the light transmission path (the larger the better), which is constrained by the size of the device.

**Ring Laser Gyro (RLG)**

In a RLG light propagates in two directions simultaneously through a gas-filled optical cavity, using three or four mirrors (see Figure 8). The RLG has excellent scale-factor stability and linearity, negligible sensitivity to acceleration, fast turn-on, excellent stability and repeatability and no moving parts.

![Figure 8. Ring Laser Gyro Schematic (Groves, 2008)](image)
RLG is a mature technology and current development efforts involve continued cost reduction rather than efforts at performance gains (Barbour, 2011).

**Fiber-Optic Gyro (FOG)**
The FOG (also known as IFOG - Interferometric Fiber-Optic Gyros) consists of a large coil of optical fiber in place of the RLG’s mirrors and optical cavity. A broadband light source is divided using beam splitters into two equal portions that are then sent through a fiber-optic coil, which can be from 100 m to 3 km in length, in opposite directions. When the beams exit the fiber they are combined. The phase shift introduced due to the Sagnac effect causes the beams to interfere. The result is a combined beam whose intensity depends on the rotation rate. Consequently, it is possible to measure the angular velocity by measuring the intensity of the combined beam. The FOG has some advantages over the RLG in that:

- the light source does not require high voltage
- has the potential for lower cost and lighter weight

A unique feature of the FOG is the ability to scale performance up and down, e.g. doubling the coil length will decrease ARW by a factor of 2. (ARW, angle random walk, is a noise specification, in units of deg/h, that is directly applicable to angle calculations.) However, unlike the RLG, the open-loop IFOG is limited in dynamic range and only has moderate scale-factor stability.

The FOG is mature technology with performance and size comparable to the RLG. However, on-going developments in solid-state optics and fiber technology could potentially lead to 0.001 deg/h performance in a miniature design (Barbour, 2011). Specifically, the development of photonic crystal fibers (PCF) and monolithic Integrated Optical Chips could lead to the development of a Miniature FOG (MFOG).

Another technology suitable for miniaturizing the FOG is the Resonant FOG (RFOG). It utilizes short lengths of fiber in which the clockwise and counter clockwise light beams are kept in resonance, which requires a very narrow-band light source and low loss fibers. RFOGs offer the potential for equivalent IFOG performance, but with coil lengths up to 100 times shorter.

**Integrated Optics Gyro (IOG)**
The RFOG architecture can be implemented in an Integrated Optics Gyro (IOG; or optical gyro on a chip). The IOG is an optical waveguide based Sagnac effect gyroscope, in which two beams of light travel in opposite directions around a waveguide ring resonator in place of an optical fiber. The IOGs are fabricated on wafers, combining the capabilities of integrated optic fabrication and MEMS fabrication.

**MEMS Gyroscopes**
Despite years of development, mechanical and optical gyroscopes still have high part counts and a requirement for parts with high-precision tolerances and intricate assembly techniques. As a result they remain expensive.
The development of MEMS gyroscopic sensors emerged later than its counterpart, accelerometers, due to the fact that there was less demand from the commercial sector. In principle, most MEMS gyroscopes operate similarly to the vibrating gyroscopes. Typical MEMS gyroscopes developments can be categorized as tuning fork and resonant ring gyroscopes.

A vibratory gyroscope comprises an element that is driven to undergo simple harmonic motion. The vibrating element may be a string, beam, pair of beams, tuning fork, ring, cylinder, or hemisphere. All operate on the same principle, which is to detect the Coriolis acceleration of the vibrating element when the gyro is rotated. To illustrate the principle, in a tuning fork gyro a proof mass attached to springs is forced to oscillate in the horizontal plane (Figure 9). A voltage is applied to a sensing electrode (sense plate) below the proof mass, creating an electrical field. The Coriolis force imparted by angular rotation causes the proof mass to oscillate in the vertical direction, which, in turn, changes the gap between the proof mass and the sense plate. This generates an AC current with amplitude proportional to the rotation rate.

![Figure 9. MEMS tuning fork gyroscope (Weber et al., 2004)](image)

Most vibratory gyros are so-called microelectromechanical system (MEMS) inertial sensors. They are built using silicon micro-machining techniques and have low part counts (a MEMS gyroscope can consist of as few as three parts) and are relatively cheap to manufacture.

At present, MEMS sensors cannot match the accuracy of optical devices; however, they are expected to do in the future. The main disadvantage of the MEMS device is that it is far less accurate, as indicated by the bias stability and angular random walk measurements.

For inertial MEMS systems, attaining suitable gyro performance is more difficult to achieve than accelerometer performance. The performance of MEMS gyroscopes is currently around 5-30 deg/h.

**Accelerometers**

Inertial navigation/surveying relies upon the measurement of acceleration which can be integrated successively to provide estimates of changes in velocity and position. Measurements of acceleration are used in preference to direct measurements of velocity because velocity and position measurements require an external reference whilst acceleration can be measured internally.
An accelerometer can be broadly classified as either a mechanical or solid state device. In this section these two types of accelerometers are described, as are MEMS accelerometers.

**Mechanical Accelerometers**

In its simplest form, the mechanical accelerometer contains a proof mass connected via a spring to the case of the instrument, as shown in Figure 10. When the case of the instrument is subjected to acceleration along its sensitive axis, the proof mass tends to resist the change in movement owing to its own inertia. As a result, the mass is displaced with respect to the body. The displacement of the mass is measured using a displacement pick-off, giving a signal that is proportional to the force $F$ acting on the mass in the direction of the input axis. Newton’s second law $F = ma$ is then used to calculate the acceleration acting on the device.

![Figure 10. Mechanical accelerometer (Woodman, 2007)](image)

Many mechanical devices commonly used in present day inertial navigation systems operate in a manner analogous to the simple spring and mass described above. In order to carry out the full navigation function, information is required about the translational motion along three axes. Commonly, three single-axis accelerometers are used to provide independent measures of specific force, although multi-axis instruments can be used. It is common practice to mount the three accelerometers with their sensitive axes mutually orthogonal, although such a configuration is not essential (Titterton and Weston, 2004).

**Solid State Accelerometers**

During the last decade, there has been intensive research effort to investigate various phenomena that could be used to produce a solid-state accelerometer. Various devices have been demonstrated, with surface acoustic wave, silicon and quartz devices being the most successful. These sensors are small, rugged, reliable and offer the characteristics needed for strapdown applications. An example of a solid-state accelerometer is the surface acoustic wave (SAW) accelerometer. A SAW accelerometer consists of a beam resonating at a particular frequency, as shown in Figure 11. The beam is anchored at only one
end. At the other end of the lever beam, a mass is attached which is free to move. When acceleration is applied along the input axis the beam bends. This causes the frequency of the surface acoustic wave to change proportionally to the applied strain. Comparison of this change with the reference frequency provides a direct measure of the acceleration applied along the sensitive axis.

Figure 11. A surface acoustic wave accelerometer (Woodman, 2007)

**MEMS Accelerometers**

Microelectromechanical silicon accelerometers use the same principles as the mechanical and solid state sensors. Current MEMS based accelerometers developments are typically divided into two types:

- pendulous; a mechanical sensor using silicon components measuring the displacement of a supported mass manufactured using MEMS techniques;
- vibrating beam; a sensor measuring the change in frequency of a vibrating element caused by a change of tension, as in SAW accelerometers.

Generally, vibrating beam devices or resonant sensors are more accurate than the pendulous type, where it is able to provide accuracy up to 1µg compared to 25µg to 1mg for pendulous accelerometers (Titterton and Weston, 2004).

The advantages of MEMS are that they are small, light and have low power consumption and short start-up times. Their main disadvantage is that they are not currently as accurate as accelerometers manufactured using traditional techniques, although the performance of MEMS devices is improving rapidly.

**All-accelerometer Navigation**

Due to the difficulty in producing high performing small gyros, efforts have been done to develop all-accelerometer systems (also known as gyro-free). Two approaches are typically used. In the first approach, also known as the ‘direct’ approach, the accelerometers are placed in fixed locations and used to measure angular acceleration (Zorn, 2002). In the second approach, the Coriolis effect is exploited and typically three opposing pairs of MEMS accelerometers are
dithered on a vibrating structure (or rotated) (Barbour, 2011). This approach allows the detection of angular rate. In both approaches, the accelerometers also measure linear acceleration to provide the full navigation solution. However, in the direct approach, the need to make one more integration step makes it more vulnerable to bias variations and noise, so the output errors grow by an order of magnitude faster over time than a conventional IMU. To date, only systems of the second kind have been reduced to practice. However, these devices only provide tactical grade performance, and are most useful in GNSS-aided applications. Techniques concerning the number of accelerometers and their specific placements continue to be studied for the direct approach. However, as noted in Zorn (2002), the concept of a navigation grade all-accelerometer IMU requires accelerometers with accuracies on the order of nano-g’s or better, and with large separation distances. Therefore the use of all-accelerometer navigation for GNSS-unavailable environments will not be viable until the far future, if ever.

**Cold Atom Sensors**

A potentially promising technology is inertial sensing based upon the atom interferometry (also known as cold atom sensors or atom optic (AO) sensors) (Barbour, 2011). AO sensors exploit the wavelike properties of atoms to achieve high sensitivities to inertial forces with accuracies orders-of-magnitude superior to conventional sensors.

As with light, the atom matter waves can be split and recombined to yield interference patterns, and thus be used to perform sensitive interferometry measurements. Laser cooling slows down the atoms so that they will have more time to experience the effects of angular rate or acceleration, and so have greater separation upon recombination. A light-pulse atom interferometer effectively measures the atoms’ inertial trajectories relative to an optical reference mirror fixed to the sensor case.

In theory, this means that the atom interferometers could make the most accurate gyroscopes, accelerometers, gravity gradiometers, and precision clocks, by orders of magnitude (Young et al., 2012). Given their extreme accuracies and sensitivities cold atom sensors are expected to revolutionize the inertial navigation.

**Supported Inertial Navigation: Integration of IMU with Other Sensors**

In the previous sections we have demonstrated that the drift is a major problem affecting INS. This section provides a brief overview of methods which can be used to reduce or correct the drift. Such methods generally fall into one of two categories, the use of sensor fusion and the application of domain specific assumptions (Woodman, 2007).

Sensor fusion can be described as the fusion of data from multiple sensors with the purpose of filtering out many of the shortcomings with the respective sensors. By fusing and weighting the data from different sensors the integrated
positioning system will achieve a more robust, reliable and accurate positioning system compared to what each individual sensors can offer.

The Kalman filter is the most popular estimation tool in the fusion of GNSS and INS data because it is optimal in theory. However, when attempting to integrate multiple positioning sensors and systems the problem to solve tend to become non-linear (the dynamics) and the noise is non-Gaussian. Furthermore, the more widely low cost IMU is adopted, the more obvious the limitations of the Kalman filter become. In recent years, there have been some successful Artificial Intelligence-based methods, such as artificial neural networks or adaptive neural fuzzy information systems, applied to GNSS/INS integration, but it is just a beginning. When it comes to state estimation for non-linear systems, there is no single solution available that clearly outperforms all other strategies. In the section below, a brief review of the most common integration architectures are given.

**GNSS/INS Integration Architecture**

The design process for GNSS/INS integration includes the trade-off between performance and cost, and cost can be strongly influenced by the extent to which GNSS/INS integration requires some modification of the inner workings of the GNSS receiver or INS. Several approaches can be used to integrate GNSS and INS systems, differing by the “depth” of the interaction and for the shared information between the systems. The more common strategies are listed below (Petovello 2003):

- Uncoupled Integration
- Loosely Coupled Integration
- Tightly Coupled Integration
- Deep or Ultra-Tight Integration

In the **uncoupled** approach, the systems work independently providing two distinct navigation solutions; usually GNSS is considered the more accurate one and is adopted when available as system solution. Moreover, the GNSS solution is used to correct (or reset) the INS solution, but without estimating the causes of the sensor drift (as happens in the other integration approaches). In absence of GNSS data the system solution is entirely supplied by the inertial sensor, which tends to drift rapidly according to its grade. For this limitation the uncoupled strategy is not commonly used.

In the **loosely coupled** approach (only) the standard outputs of the GNSS receiver and INS are used as inputs to a sensor system integrating filter (often a Kalman filter), the outputs of which are the estimates of “navigation variables” (including position and velocity) based on both subsystem outputs. Although each subsystem (GNSS and INS) may already include its own Kalman filter, the integration architecture does not modify it in any way. There are many variants of the loose integration; however, the key feature is that both the INS and GNSS receiver are independent navigators. The information from the two is blended to form a third navigation solution.
Many GNSS/INS integrations are loosely coupled, giving up a great deal of performance in return for simplicity of integration. Using this principle the position and velocity estimates of a GNSS receiver are used as observations in an INS filter for estimation of INS errors, a reduction of GNSS-noise and a bridging of GNSS outages are possible (see Figure 12).

Loosely-coupled integration is characterized by the following features (Abdel-Hamid, 2005; Gebre-Egziabher, 2007):

- independent generation of navigation solutions (position, velocity, attitude) by both the GNSS and the INS alone; the information from the two is subsequently combined in the KF to provide filtered results
- the differences between the GNSS and inertial solutions are fed back in form of estimated errors to carry out the recalibration
- state vector is smaller when compared to tightly-coupled integration allowing faster signal processing and lower demands on the hardware
- it is usually implemented with higher quality inertial sensors (navigation or tactical grade) if the GNSS outages are expected to be long in duration
- lower quality inertial sensors (consumer or automotive grade) are suited for applications where GNSS outages are infrequent and short in duration
- the main disadvantage is the cascade realization of the filters that causes the measurement error to possibly become time correlated; hence, allowing only suboptimal solution.
The more tightly coupled implementations use less standard subsystem outputs, such as pseudoranges from GNSS receivers or raw accelerations from inertial navigators (see Figure 13). These outputs generally require software changes within “standalone” GNSS receivers or INSs, and may even require hardware changes. The filter model used for system integration may also include variables such as GNSS signal propagation delays or accelerometer scale factor errors, and the estimated values of these variables may be used in the internal implementations of the GNSS receiver or INS. Tightly coupled implementations may impact the internal inputs within the GNSS receiver or INS, as well. The acceleration outputs from the INS can be used to tighten the GNSS Doppler tracking loops, and the position estimates from the INS can be used for faster re-acquisition after GNSS outages. Also, the INS accelerometers and gyroscopes can be recalibrated in real time to improve free-inertial performance during GNSS signal outages. The tightly coupled approach benefits from GNSS measurement updates even if there are less than four satellites available for a complete GNSS solution. Loose integration, however, has the advantage of redundancy because the INS and GNSS receiver still produce independent solutions.

Tightly–coupled system integration has the following features (Abdel-Hamid, 2005; Gebre-Egziabher, 2007):

- GNSS system provides pseudoranges, Doppler or carrier phase measurements that are combined directly with the navigation solution provided by inertial sensors
- real-time feedback of INS velocities to the GNSS receiver enables an accurate prediction of GNSS pseudorange and phase at the following epoch, thus allowing a smaller bandwidth of the receiver’s tracking loop in a high-dynamic environment with subsequent increase in accuracy
- more complex concept resulting in larger state vector and hence higher hardware demands
• less sub-optimal (and therefore more accurate) compared to the loosely-coupled integration because the basic GNSS observables are not possibly as correlated as the position and velocity solutions used in the loosely-coupled integration

• offers possibility to implement fault detection and isolation scheme for verification of the quality of pseudorange or Doppler measurements.

The most recent research activity is an approach called deep integration. In this case, the integration of the GNSS and INS devices no longer works as independent systems. GNSS measurements are used to estimate INS errors and INS measurements to aid GNSS receiver tracking loops; this integration is clearly at deep level and requires access to receiver’s firmware and so is usually implemented by receiver manufacturers or with software receivers (Gautier, 2003). One of the many advantages of the deep integration architecture is that it enhances the robustness of GNSS to interference and jamming.

In summary, by going from the loose to the deep integration architectures, we gain robustness to GNSS outages either due to dynamics, interference, or jamming. However, this increased robustness comes at the sacrifice of system simplicity, redundancy, and independence of the INS and GNSS receiver.

Augmentation Sensors

From the previous discussion it is clear that a standalone INS cannot fulfil precision requirements for topographic surveying. No suitable inertial technology exists today, or is under development with expectations of getting close to this performance. Consequently, the use of active and passive augmentation sensors (aiding devices) are required to provide velocity and/or attitude updates to bind the error due to the drift in the inertial system. Examples of possible augmentation sensors in hand-held equipment are magnetometers and barometers. There can also be improvements from using special procedures such as ZUPTs (Zero Velocity Updates), mapping information, or path crossings. Augmentation sensors open the door to the use of much lower performing inertial sensors, so that current technology can be used.

Magnetometers

One type of sensor commonly used to reduce the drift of INS is the vector magnetometer (Woodman, 2007). Magnetometers are used to augment the heading information in integrated systems by furnishing orientation with respect to the earth’s magnetic field. IMUs often contain three orthogonal magnetometers in addition to the orthogonal gyroscopes and accelerometers. The magnetometers measure the strength and direction of the local magnetic field allowing the north direction to be found. Magnetometers are not accurate enough to replace gyroscopes in INSs. Especially, they are affected by the local disturbances in the earth’s magnetic field caused by nearby magnetic objects, e.g. iron in the ground, iron constructions, cars, computers, etc. However, their data can be fused with gyroscope data to improve the accuracy of the calculated orientation.
The limitation is not the accuracy of existing magnetometers. The largest errors are due to the deviations in the magnetic field model and these are difficult to compensate for. Future magnetometers will be smaller, cheaper and more accurate but this will probably have negligible effect on the positioning performance in at least urban areas (Rantakokko et al., 2007).

**Barometers**

The atmospheric pressure decreases with an increased height above sea level. Consequently, if we can model how the atmospheric pressure varies with altitude we can estimate the altitude by measuring the atmospheric pressure e.g. with a barometer.

However, the atmospheric pressure varies due to other phenomena such as weather and temperature. The differences in air pressure due to weather changes are relatively small during short time intervals, but the absolute error will drift unless it is regularly calibrated (Rantakokko et al., 2007). When GNSS signals are available, height determination from GNSS can be used for this calibration. The addition of a barometer to integrated navigation systems is not a novel idea as they are typically inserted in INSs to constrain position error growth in the height component. Nominal sea level atmospheric pressure is approximately 0.1 MPa and decreases exponentially with increasing altitude at a rate of approximately 10 Pa/m (Hopkins et al., 2010). Hence, barometer sensitivity on the order of ~1 Pa enables sub-meter altitude resolution.

In order to reach accuracies below 3 dm (1σ) the limitation is not the accuracy of existing barometers. The largest uncertainties are due to the variations of the atmospheric pressure which are hard to model and to account for.

**Digital Camera**

Digital camera can be used also as a device for measurement of direction towards object, which means that it is possible to derive the change in camera orientation from sequence of images. This is a classical problem within the field of computer vision and there are many mathematical formulations for its solution, see e.g. Faugeras and Maybank (1990) or Harris (1987). The cameras can be used as a supporting sensor for inertial navigation (see e.g. Horemuz and Gajdamowicz 2005 or Oskiper 2012). Using camera in addition to GNSS can be the only reasonable solution for use of MMS in indoor and urban environments.

**Domain Specific Assumptions**

In some applications it is possible to make assumptions about the movement of the body to which the IMU is attached. Such assumptions can be used to minimize drift. One example in which domain specific assumptions are exploited is a shoe mounted IMU (Schmidt and Phillips, 2010), which is used to track the location of a pedestrian. The assumption that a pedestrian’s foot has zero velocity when in contact with the ground is used to provide zero-velocity updates, allowing drift in velocity to be periodically corrected. By measuring the acceleration due to gravity when the device is stationary it is also possible to estimate and make adjustments for the tilt of the device.
This might be exploited in hand-held equipment by mounting the IMU at the bottom of a measuring pole, used by the land surveyor. The main disadvantage of using domain specific assumptions is that the assumptions must hold for the results to be valid. For instance, in the case of the foot mounted IMU this would fail if the pedestrian would use an escalator. The benefits obtained from using assumptions must be weighed against the risk that they may be broken.

Uncertainty Analysis

The data delivered by MMS is point cloud of the area of interest. Each point in the point cloud has 3D coordinates expressed in a required reference system and eventually another attributes like intensity of returned signal or colour. The quality of data depends on many factors. The term “quality” usually refers to completeness and accuracy of data. In this report we focus only on the accuracy aspect of the quality. The term “accuracy” depicts the degree of conformance between the “true” and measured quantity (coordinates, velocity and orientation). It is possible to assess the accuracy analytically, i.e. to compute the expected accuracy taking into account the contribution from all possible error sources. In this case we do not know the “true” values, so we refer to this analysis as uncertainty, or precision analysis. The result of the analysis is expected standard uncertainty (expressed in term of standard deviation) of the determined quantities, in our case the coordinates, velocity and orientation of mapping sensor and coordinates of mapped points.

There are three groups of uncertainty sources, which affect the accuracy of each individual point in the point cloud:

- Uncertainty from positioning module, i.e. errors stemming from gyroscopes, accelerometers, GNSS and processing errors
- Uncertainty from mapping sensor, i.e. errors from laser scanner or cameras
- Environmental and calibration uncertainties, i.e. atmospheric influence and uncertainty in knowledge of the offset and relative orientation between IMU and mapping sensors and IMU and GNSS antenna

Uncertainty from Positioning Module

The positioning module includes inertial and GNSS sensors. In this analysis we assume that the GNSS receiver is of geodetic type and there are at least 4 satellites with good geometry available, i.e. the receiver delivers coordinates with standard uncertainty of 15 mm horizontal and 30 mm vertical every second. We also assume that the IMU and GNSS are correctly synchronised, i.e. the uncertainty in synchronisation is negligibly small. If this is not true, then both accuracy in position and orientation would be compromised, especially during the vehicle’s manoeuvres (turning or acceleration). But in the case of correct synchronisation, the uncertainty in position delivered by the positioning module
depends mainly on the uncertainty of GNSS coordinates. The increase in positional uncertainty between GNSS updates, i.e. during 1 s interval is negligible. This claim can be supported by example in equation (4): if we use \( t = 1 \) s we get positional error of 0.3 mm due to error in gyro. Positional error due to accelerometer bias = 50 \( \mu \)g during 1 s would be 0.25 mm (using equation in Table 2.) The uncertainty in the orientation depends mainly on the quality of inertial sensors (both gyroscopes and accelerometers). We should point out that GNSS updates make it possible to improve also accuracy in orientation, i.e. determination of tilt (roll, pitch) and heading (yaw) of the platform. But the heading can be improved only when the platform is accelerating, i.e. turning or changing speed. So if the vehicle is static or in constant velocity, GNSS does not help to estimate the heading, i.e. its uncertainty increase depends on performance of the gyroscopes. That is why the uncertainty in heading is slightly higher than in tilt. Table 4 shows summary of achievable standard uncertainties in the orientation for GNSS aided tactical and navigational grade INS. Update interval in the first column is time between GNSS updates. Values in parentheses are computed for concrete instruments manufactured by Honeywell, using analytical method described in Horemuz and Sjöberg (2001).

Table 4. Inertial attitude determination performance with GNSS aiding (Skaloud 2002).

<table>
<thead>
<tr>
<th>Update interval</th>
<th>Navigational grade (usually RLG)</th>
<th>Tactical grade (usually FOG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sec</td>
<td>0.0008 – 0.0014 (0.0002)²</td>
<td>0.0008 – 0.0020</td>
</tr>
<tr>
<td>1 – 3 min</td>
<td>0.0014 – 0.0030</td>
<td>0.0040 – 0.0050</td>
</tr>
</tbody>
</table>

Table 5. Uncertainty of attitude determination with GNSS aiding.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Automotive (usually MEMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>roll, pitch [deg]</td>
<td>yaw [deg]</td>
</tr>
<tr>
<td>Specifications⁴</td>
<td>0.2</td>
</tr>
<tr>
<td>Analytical</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Input to the analytical computation are only the uncertainty values shown in the footnote and a suitable satellite configuration leading to 1 cm uncertainty in horizontal coordinates and 2 cm in height. The analytically computed

² Honeywell HG9900, ARW = 0.002 ° / \( \sqrt{t} \), VRW = 0.000025 \( m / \sqrt{s} \)
³ Honeywell HG1700, ARW = 0.2 ° / \( \sqrt{t} \), VRW = 0.003 \( m / \sqrt{s} \)
⁴ Spatial, ARW = 0.3 ° / \( \sqrt{t} \), 0.004 \( m / \sqrt{s} \), 100 s correlation time
uncertainties are slightly more optimistic than those published by Skaloud (2002), which is expected, since the analytical computations take into account only the sensor noise and ideal GNSS observations. In practice, we will never get these ideal conditions and the results will be affected by other error sources, namely unmodelled GNSS systematic effects (mainly multipath and atmosphere) and errors coming from processing algorithm limitations. These limitations are caused by approximations that are necessary to apply when integrating the output from gyroscopes and accelerometers. Another cause of these limitations is the fact that the inertial sensors do not measure the acceleration and rotation rate continuously, but discretely, usually with frequency 100 – 300 Hz. These limitations can cause errors in both position and orientation and they are especially pronounced in high dynamics environment, e.g. sudden manoeuvres or vibrations caused by vehicle’s engine. For completeness we also show uncertainty for automotive grade IMU in Table 5.

It should also be pointed out that all above given uncertainties are determined for post processing, i.e. data from whole measuring session were processed in a smoothing filter, which means that position and orientation for any given time epoch were computed using measured data from both sides of the epoch. On contrary, in case of real time processing, only data up to the given epoch are available and the results are less accurate compared to post processing.

As an example of commercial positioning module we can mention Applanix POS LV 210 – see Table 6.

Table 6. Uncertainty specifications for Applanix POS LV 210.

<table>
<thead>
<tr>
<th></th>
<th>Post processing</th>
<th>RTK</th>
<th>DGNSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal position (m)</td>
<td>0.020</td>
<td>0.035</td>
<td>0.300</td>
</tr>
<tr>
<td>Height (m)</td>
<td>0.050</td>
<td>0.050</td>
<td>0.500</td>
</tr>
<tr>
<td>Roll and Pitch (°)</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>True Heading (°)</td>
<td>0.050</td>
<td>0.100</td>
<td>0.200</td>
</tr>
</tbody>
</table>

Uncertainty from Mapping Sensors

The most common mapping sensors used in MMS are laser scanners and digital cameras. Their contribution to the uncertainty in the coordinates of points is small if the distance between MMS and the measured point is short but it increases with increasing distance. The increase is approximately quadratic for cameras and linear for laser scanner. Using a stereo pair of cameras, the positional uncertainty of measured points is on cm level for distances up to 10 – 20 m, on dm level for distances 20 – 60 m and on metre level for longer distances. Typical uncertainty for laser scanning instrument is 5 – 10 mm in distance measurement and 0.001 – 0.005° for angular measurement. As an example we can mention instrument Riegl VQ450, which is constructed for use in MMS; according to the specification its standard uncertainty in distance
measurement is 8 mm and 0.001° in angular measurement, which would cause transversal error in point’s position (error in the direction of scanning motion of the laser beam) 0.2 mm on 10 m distance or 2 mm on 100 m distance. From this example we can deduce that the main uncertainty contribution from laser scanner for shorter distances is the distance measurement. The uncertainty in angular measurement becomes significant (on cm level) in airborne applications.

Environmental and Calibration Uncertainties

Atmosphere
The atmosphere is propagation media for GNSS signals, as well as for the laser beam from scanning instrument and hence it influences the distance measurements. In case of GNSS, we can distinguish between ionospheric and tropospheric delay. Both delays must be properly accounted for when processing the GNSS observations, which is routinely performed by all GNSS software packages. The uncertainties in atmospheric modelling in GNSS processing are included in the uncertainties of GNSS coordinates. Special attention should be paid in case of airborne applications, where there is significant height difference between reference and roving receivers. In this case it is mandatory to measure temperature and atmospheric pressure to be able to eliminate the tropospheric delay from GNSS observations. This is not necessary in surveying applications, where the height difference is usually small and ordinary differential processing eliminates the tropospheric influence sufficiently. If the temperature and pressure is not measured, errors on dm level could be introduced to the computed GNSS coordinates (Collins et al. 1996, mainly height is affected).

Laser scanning instruments are affected by troposphere only and they are calibrated for certain “normal” atmosphere, usually 15 °C and 1013.25 hPa. If the measurements are performed in different conditions, an atmospheric correction must be computed. The size of correction is approximately 1 mm/km per change of temperature by 1 °C or change of pressure by 1 hPa. The atmospheric correction introduces only small uncertainty into the distance measurement, especially in case of short distances. For example even if the measurements are done at 0 °C and the operator forgets to measure the temperature, the error on 100 m distance would be only 1.5 mm.

Calibration
For obvious reasons different sensors cannot occupy the same place, but there must be certain offset between them, which is often called lever arm. To be able to process data from the sensors together, the offset and mutual orientation must be known. For example, GNSS receiver determines the position of its antenna, but we need to determine also the coordinates of IMU and laser scanner. The process of determination of the lever arm is usually referred as boresight calibration or simply calibration. The calibration parameters typically include:

- Vector between GNSS antenna and IMU
- Vector between IMU and laser scanner and/or camera
• Orientation (rotation matrix) between IMU and laser scanner and or camera

The calibration parameters remain constant if the relative position and orientation between sensors have not changed. The parameters would (slightly) change after each new set up of sensors, i.e. every time the sensors are mounted onto surveying platform. They can also change due to the vibration of the platform and due to its elasticity. Therefore the calibration parameters must be determined after the sensors were mounted on the platform or preferably before each measurement session.

The parameters can be determined directly, using totals station and/or terrestrial laser scanner, or indirectly using an external calibration field consisting of pre-surveyed points or objects.

Test Methods for Positioning Modules

As we could see from the previous discussion, a MMS is a complex system with many possible error sources. The analytical study can predict the precision (or uncertainty) but cannot guarantee the accuracy of the results. This can be compared with other surveying methods, e.g. measurements with RTK or total station, where the accuracy is verified experimentally either by measuring on known points (calibration baselines) or by comparing the measurements to those performed by other instrument/methods. Similar principles can be applied also to accuracy verification of positioning modules.

There are several alternatives how to verify the performance of a GNSS/INS positioning module:

• Laboratory testing
• Field testing using an independent positioning system
• Field testing using the mapping sensors

Laboratory Testing

The individual sensors are tested and calibrated on so called rotary tables (Figure 14). The table can rotate and translate around two or three axes and the movements can be precisely programmed and controlled. In this way one can obtain “true” values of rotations and translations (and their rates) and compare them with the values obtained by sensors. There are several laboratories in the world equipped with GPS/INS test and verification facility, which consists of a rotary table and a GPS simulator generating GPS signals. GPS antenna is mounted on top of the rotary table and it receives the simulated signals. The combined GPS/IMU solution can be compared with the values obtained from the rotary table and the accuracy in both position and orientation can be evaluated.
An example of such facility is The Institute of Space Systems - DLR Bremen\(^5\). We are not aware of such laboratory in Sweden, but there are some laboratories (e.g. Imego\(^6\)) equipped with rotary tables.

![Figure 14. Rotary table. (The Institute of Space Systems - DLR Bremen)](image)

**Field Testing Using an Independent Positioning System: Direct Method**

In this approach the position and orientation of the positioning module is determined directly by an independent method. By “direct determination” we mean that the position and orientation of the platform itself is measured and “indirect determination” refers to methods where the position of surveyed objects (by MMS) is used to determine the accuracy of the positioning module. The primary requirement is that the “independent method” is more accurate than tested instrument.

The “direct method” is in principle identical to the laboratory testing described in the previous section. The main difference is that the testing is done in a production environment and that the reference (“true”) values are not determined with as high accuracy as in the laboratory testing.

The following “independent positioning systems” can be considered for direct method:

- **A positioning module with IMU of higher grade.** The tested module (using e.g. tactical grade IMU) is mounted on a vehicle (car or aircraft) along with a “reference” module using a navigational grade IMU. The difference between position and orientation delivered by these systems should be constant, depending on the offset between the modules.

- **High accuracy tilt sensors** are capable to determine tilt (roll and pitch) of a static platform. They can be used to measure roll and pitch of the platform when vehicle is not moving. The standard uncertainty of the high accuracy tilt meters is around 0.05°.

- **Total station measurements.** Two or three prisms attached to the platform can be surveyed by two or three total stations which are set up in


\(^6\) [http://www.imego.com/](http://www.imego.com/)
the given area see Figure 15. Two prisms enable heading determination; three prisms enable determination of all three orientation angles. The problem here is the uncertainty of the determined angle: if we demand small uncertainty, the distance between prisms must be sufficiently large. The uncertainty of bearing $\varphi$ computed from two surveyed points A and B is obtained from the following equation:

$$u(\varphi_{AB}) = \frac{u(\text{point})}{d_{AB}}$$

(6)

For example if we consider standard positional uncertainty of a point measured by total station $u(\text{point}) = 3$ mm, the standard uncertainty in the computed bearing (=orientation angle) would be $0.04^\circ$ for distance between points $d_{AB} = 3$ m, which is more than the angular uncertainty from tactical and navigational grade IMU. To decrease the uncertainty $d_{AB}$ must be increased, which is not always possible taking into account the length of the car.

![Figure 15. Direct method using total station measurements.](image)

- **Camera observations.** Two or more calibrated cameras are installed in the area and they are taking images synchronously. By measuring image coordinates of the positioning platform, its position and orientation can be computed. The expected standard uncertainty in photogrammetrically determined coordinates is on cm level (if the distance between cameras and vehicle is short, say tens of meters). Hence the standard uncertainty of orientation angles would be higher compared to those determined by total station.

**Field Testing Using the Mapping Sensors: Indirect Method**

In this case, the mapping sensors are attached to the positioning module and they measure the surrounding objects. Then it is possible to compare measured
coordinates of the objects with the coordinates measured by some other, preferably more accurate method/instrument. Alternatively, the objects are not measured by other method, but MMS would survey objects many times, preferably from different direction. For example one could choose a street and drive the MMS in both directions and if possible also in different distances between the car and facades. Then it would be possible to compare the coordinates of surveyed objects obtained from different runs and precision (repeatability) could be analysed. The advantage of this approach is that neither external measurements nor additional sensors are necessary. However, the accuracy analysis would not be possible, but for most application the precision is more important, since high precision guarantees that the relative position between objects and the shape of object is captured correctly.

Summary

The aim of this report was to study the possibilities of verification of positioning module's precision and accuracy. We assume that the positioning module consists of a GNSS receiver, IMU and processing software and its main purpose is to determine the trajectory (position and orientation) of mapping sensors (laser scanner and/or digital camera).

The positional accuracy of a positioning module depends almost exclusively on the GNSS positioning accuracy. IMU can bridge only short term GNSS outage (a few seconds) with reasonable (cm-level) accuracy. In case of uninterrupted GNSS positional update once per second, the resulting positional accuracy of the module is the same as accuracy of GNSS solution. E.g. if GNSS solution would jump for some reason several cm, or even dm, the combined GNSS/IMU solution would converge quickly to the GNSS solution.

The determination of orientation is more stable: the positioning module can deliver reasonably accurate orientation even if GNSS solution is not available several minutes. The accuracy of orientation depends on the quality of the inertial sensors, both accelerometers and gyroscopes. It means that high quality gyroscopes do not guarantee high accuracy of orientation if they are not combined with high accuracy accelerometers. This claim is valid for combined GNSS/IMU solution; of course for standalone INS navigation only gyroscopes determine the orientation. Expected accuracy of orientation for different grades of IMU is given in Table 4 and Table 5.

The expected accuracy is actually expected precision, since it is assumed that there are no uncalibrated and/or unmodelled systematic errors, which cannot be guaranteed in real world applications. That is why experimental test measurements are necessary to verify the actual performance of positioning module. There are several alternatives of performing test measurements. If the aim of the test is to verify accuracy, external and independent measurements are required to determine “true” or reference values of the position and orientation of the platform or “true” coordinates of objects captured by the MMS.
The most rigorous tests can be performed in laboratory, using rotary table and GNSS simulator, which can provide the reference values of position and orientation with the best possible accuracy. Disadvantage of laboratory test is that the positioning module is not mounted in the same way as during normal operation and the tests can be performed only in limited range of dynamics. Another limitation of such test is the availability (only a few laboratories in the world) and high cost.

The tests based on field observations are more feasible. The direct methods allow for accuracy determination, but they require extra sensors and or surveying equipment. It is also problematic to reach significantly better accuracy in the determination of reference values, which is essential for accuracy assessment. Therefore, if for given applications, the verification of the positioning module is not as important as the verification of whole MMS, the most suitable are the indirect methods, where the mapping sensors attached to the positioning module are used to survey several test objects. If the coordinates of the test objects are known then accuracy of MMS can be evaluated; otherwise precision can be determined from repeated observations of the same objects.

In the following work within the project we will evaluate more in detail different verification methods.
References


